Oberseminar

Block Theory

SS 2019

Termin: **Mo. 15:30 – 17:00 (Raum 48-438)** Beginn: 29.04.19

29.04.19	Gunter Malle	**I. Intro. to block theory via algebras and modules**
06.05.19	Emil Rotilio	**II. Intro. to block theory via character theory I**
13.05.19	Anantha Prasad Subbaraya	**III. Intro. to block theory via character theory II**
20.05.19	Giovanni De Franceschi	**IV. Clifford theory of blocks**
27.05.19	Alessandro Paolini	**VII. Blocks of finite groups of Lie type**
03.06.19	Ruwen Hollenbach	**VI. Blocks of p-solvable groups**
17.06.19	Bernhard Böhmler	**X. Blocks with cyclic defect I**
24.06.19	Michael Livesey	**VIII. On Donovan's conjecture for abelian defect groups and strong Frobenius numbers**
01.07.19	Niamh Farrell	**IX. Nilpotent blocks**
08.07.19	Shigeo Koshitani	**V. Equivalences of blocks algebras**
18.07.19*	Caroline Lassueur	**XI. Blocks with cyclic defect II**

*Room 48-436, 16:30-18:00

Literatur

[Alp86] J. L. Alperin, *Local representation theory*, Cambridge Studies in Advanced Mathematics, vol. 11, Cambridge University Press, Cambridge, 1986, Modular representations as an introduction to the local representation theory of finite groups. MR 860771 (87i:20002)

[BMM93] Michel Broué, Gunter Malle, and Jean Michel, *Generic blocks of finite reductive groups*, Astérisque (1993), no. 212, 7–92. MR 1235832 (95d:20072)

[BR03] Cédric Bonnafé and Raphaël Rouquier, *Catégories dérivées et variétés de Deligne-Lusztig*, Publ. Math. Inst. Hautes Études Sci. (2003), no. 97, 1–59. MR 2010739

[CE93] Marc Cabanes and Michel Enguehard, *Unipotent blocks of finite reductive groups of a given type*, Math. Z. **213** (1993), no. 3, 479–490. MR 1227495 (94h:20048)

[CE99] ______, *On blocks of finite reductive groups and twisted induction*, Adv. Math. **145** (1999), no. 2, 189–229. MR 1704575

[Eng00] Michel Enguehard, *Sur les ℓ-blocs unipotents des groupes réductifs finis quand ℓ est mauvais*, J. Algebra **230** (2000), no. 2, 334–377. MR 1775796 (2001i:20089)

[FS82] Paul Fong and Bhama Srinivasan, *The blocks of finite general linear and unitary groups*, Invent. Math. **69** (1982), no. 1, 109–153. MR 671655

[FS89] ______, *The blocks of finite classical groups*, J. Reine Angew. Math. **396** (1989), 122–191. MR 988550

[KM13] Radha Kessar and Gunter Malle, *Quasi-isolated blocks and Brauer's height zero conjecture*, Ann. of Math. (2) **178** (2013), no. 1, 321–384. MR 3043583

[KM15] ______, *Lusztig induction and ℓ-blocks of finite reductive groups*, Pacific J. Math. **279** (2015), no. 1-2, 269–298. MR 3437779

[Lin18] Markus Linckelmann, *The block theory of finite group algebras. Vol. II*, London Mathematical Society Student Texts, vol. 92, Cambridge University Press, Cambridge, 2018. MR 3821517

[LP10] Klaus Lux and Herbert Pahlings, *Representations of groups*, Cambridge Studies in Advanced Mathematics, vol. 124, Cambridge University Press, Cambridge, 2010, A computational approach. MR 2680716

[Nav98] G. Navarro, *Characters and blocks of finite groups*, London Mathematical Society Lecture Note Series, vol. 250, Cambridge University Press, Cambridge, 1998. MR 1632299

[NT89] Hirosi Nagao and Yukio Tsushima, *Representations of finite groups*, Academic Press, Inc., Boston, MA, 1989. MR 998775 (90h:20008)

[Sch85] Klaus-Dieter Schewe, *Blöcke exzeptioneller Chevalley-Gruppen*, Bonner Mathematische Schriften [Bonn Mathematical Publications], vol. 165, Universität Bonn, Mathematisches Institut, Bonn, 1985, Dissertation, Rheinische Friedrich-Wilhelm-Universität, Bonn, 1985. MR 825237

[Web16] Peter Webb, *A course in finite group representation theory*, Cambridge Studies in Advanced Mathematics, vol. 161, Cambridge University Press, Cambridge, 2016. MR 3617363

Interessierte Hörer sowie weitere Vortragende sind herzlich willkommen!

Talk Descriptions (tentative)

Talk length: 90 minutes

29.04.19 – TALK I. Introduction to block theory via algebras and modules
The aim of this talk is to introduce block theory from the point of view of modules and algebras including: a recap of the essential results needed from representation theory, central primitive idempotents, blocks, defect groups, Brauer's main theorems, the Green correspondence etc. Sources: [Alp86], [Web16], [LP10], [NT89]

06.05.19 – TALK II. Introduction to block theory via character theory I
The aim of this talk is to introduce blocks from a character theoretic point of view. This includes an introduction to Brauer characters and decomposition numbers. In doing this we will also fix some notation from [Nav98], so that later talks can be presented using this notation if it is more suitable than the notation from Talk I. Source: [Nav98]

Chapter 1: all assumed
Chapter 2: (include/omit things depending on what was included in Talk I)

- Definitions: Brauer character, trivial Brauer character, decomposition numbers/matrices, projective indecomposable character
- Properties of Brauer characters - Lemma 2.2, Theorem 2.3, Corollary 2.9, Theorem 2.12, Corollary 2.16
- Properties of decomposition matrices - Corollary 2.11
- Properties of projective indecomposable characters - Corollary 2.17

Chapter 3: Up to Definition 3.1 (p-block)

13.05.19 – TALK III. Introduction to block theory via character theory II
A continuation of Talk II - we will show that the character theoretic characterization of a block is the same as the module theoretic definition from Talk I, and discuss some properties of blocks relating to their defect groups. Source: [Nav98]

Chapter 3:

- Recall the character theoretic definition of a block (Definition 3.1)
- Theorem 3.3: relate decomposition numbers to new definition of a block
- Define Linked, Brauer graph (Definition 3.4)
- Theorem 3.9: characterizations of the ordinary characters in a p-block via connected components of Brauer graphs
- Lemma 3.13: relate character theoretic approach to algebra/module theoretic approach
- Define defect of a block (Definition 3.15)
- If time permits: Properties relating to defect (Corollary 3.17, Theorem 3.18)

20.05.19 – TALK IV. Clifford theory of blocks

1) **Defect groups.** Recall the definition of the defect group of a block from Talk I and then briefly give a definition from Navarro (either Theorem 4.3, OR Corollary 4.5, OR introduce Brauer homomorphism and use Theorem 4.11), and then state Theorem 4.6. Make sure to mention notations $\delta(B)$ and D_B.
2) **Clifford theory of blocks.** Follow one or both of the sources [Nav98, Chapter 9], [NT89, Chapter 5, Sections 5, 8].
   - Describe action of G on the set of blocks of a normal subgroup N, define the stabilizer/inertial group of a block $T(b)$, define a **covering block**. [Nav98, page 193]
   - Maybe [Nav98, Theorem 9.1] (state+idea of proof)
   - [Nav98, Theorem 9.2] ([NT89, Lemma 5.7])
   - [Nav98, Corollary 9.3] ([NT89, Lemma 5.3 second part])
   - [Nav98, Theorem 9.4] ([NT89, Lemma 5.8 (ii)])
   - [Nav98, Theorem 9.5] ([NT89, Theorem 5.5])
   - [Nav98, Corollary 9.6] ([NT89, Theorem 5.6])
   - **Fong-Reynolds** [Nav98, Theorem 9.14] ([NT89, Theorem 5.10])
   - **Knörr** [Nav98, Theorem 9.26] ([NT89, Theorem 5.10])
   - Define **dominated block** ([Nav98, page 198] ([NT89, page 360 + Lemma 8.4 and 8.6]))
   - [Nav98, Theorem 9.9] ([NT89, Theorem 8.8])
   - [Nav98, Theorem 9.10] ([NT89, Theorem 8.11]). Point out what this Theorem implies if the normal p-subgroup mentioned is central.

27.05.19– TALK V. Blocks of finite groups of Lie type

This survey talk will present the theory of blocks of finite groups of Lie type, including the parametrization of blocks by e-Jordan cuspidal pairs. Sources: [CE93], [CE99], [KM15] etc

- Most general statement of parametrization: [KM15, Theorem A]
- Quasi-isolated blocks of exceptional groups in bad primes: [KM13, Theorem 1.2]
- Reduction to quasi-isolated blocks: follows from [BR03, Theoreme B']
- Arbitrary blocks for $\ell \geq 7$: [CE99, Theorem]
- Unipotent blocks in bad primes: [Eng00, Theoreme A, E.bis]
- Unipotent blocks for good primes: [CE93, Theorem]
- Unipotent blocks for large primes: [BMM93, Fundamental Theorem 3.2]
- Certain blocks of exceptional groups of Lie type: [Sch85]
- Classical groups: [FS82], [FS89]

03.06.19 – TALK VI. Blocks of p-solvable groups

This talk will present the case of blocks of p-solvable groups – what is known and why this situation works so nicely. Source: [Nav98, Chapter 10]

17.06.19 – TALK VII. Blocks with cyclic defect I
In this talk we will revisit the properties of blocks with cyclic defect which we saw in the Oberseminar last semester in more detail. Source: [Alp86, Chapter V].

24.06.19 – TALK VIII. On Donovan's conjecture for abelian defect groups and strong Frobenius numbers
Abstract: Donovan's conjecture states that for a fixed p-group P there are only finitely many blocks (up to Morita equivalence) with defect group isomorphic to P. In this talk I will talk about recent progress made on this question for P abelian. The main tool used is the strong Frobenius number of a block. I will also talk about the corresponding results over an appropriately defined complete, discrete valuation ring. This is joint work with Charles Eaton and Florian Eisele.

01.07.19 – TALK IX. Nilpotent Blocks
In this survey talk we will define nilpotent blocks and present the main results known about these blocks. Source: [Lin18]

08.07.19 – TALK X. Equivalences of block algebras

18.07.19 16:30 Room 48-436– TALK XI. Blocks with cyclic defect II
In this talk we will look at the indecomposable modules of blocks with cyclic defect.